MCPC Editorial

Jackson Goerner

October 2, 2021

Some stats...

- 15 teams
- 800 submissions
- 15 external teams

Triangle It! - 3 Points

Author: Jackson, Monash: 13, External: 12.

Realisation

If $a^2 + b^2 = c^2$ for some integers, we can solve for each value:

$$a = \sqrt{c^2 - b^2}$$
$$b = \sqrt{c^2 - a^2}$$
$$c = \sqrt{a^2 + b^2}$$

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So solution checks if the sum or difference of our two given numbers is a **non-zero** square number.

Travelling Salesman - 3 Points

Author: Jackson, Monash: 11, External: 10.

Need to check two things:

- Movements need to line up: $a\ x$ then $x\ b$
- Every position shows up twice (Ensures no double loops)

Meow - 3 Points

Author: Jackson, Monash: 9, External: 9.

You can write code to step through the string and check rules, or you can use regex!

```
import re

meow = re.compile("^me+ow+$")
purr = re.compile("^purr+$")
roar = re.compile("^ro+ar$")

if meow.match(s) is not None:
# ...
```

Comparing Hands - 3 Points

Author: Jackson, Monash: 2, External: 5.

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Gotcha case

King, Ace, Two, Three, Four is a straight.
So you can't just choose the lowest value card and see if the next 4 cards are 1 higher.

Government Divided - Part 1 - 10 Points

Author: Jackson, Monash: 10, External: 11.

Realisation

The total number of factors of x is the number of ways work can be divided.

If the prime factorisation of x is $a_1^{b_1} a_2^{b_2} \cdots$, with $b_i \geq 1$, then the total number of factors f is

$$f = (b_1 + 1) \times (b_2 + 1) \times \cdots$$

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Calculating the prime factorisation of many numbers quickly can be done with the Sieve of Eratosthenes.

Guess Who? - 10 Points

Author: Jackson, Monash: 5, External: 5.

Realisation

For every feature, if we make a query for all but 1 of the options, we can intuit the option that our candidate has (If one of the queries comes back positive then that, otherwise the one option you didn't query).
Doing this for each feature only requires at most 18 queries.

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Doing this for each feature only requires at most 18 queries.

This can also be done by trying to play guess who optimally (Making the query that will split the crowd in approximately half).

Difficulty Curve - 10 Points

Author: Jackson, Monash: 2, External: 3.

Realisation

If there is a difficulty difference $d_i - d_j = c$, and we want the maximum smoothness to be p, then the number of "filler" problems required will be

$$\left\lfloor \frac{c-1}{p} \right\rfloor .$$

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$$\left\lfloor \frac{c-1}{p} \right\rfloor.$$

So binary search to find the smallest p for which k additions will suffice. (Also some very cheeky edge cases here)

Find and Replace - 10 Points

Author: Jackson, Monash: 9, External: 10.

Realisation

The string will repeat forever if and only if:

- "professor" is part of the Professor's name.
- "professor" is in the essay at least once.

Otherwise, we can just simulate.

Infinite Money - 30 Points

Author: Jackson, Monash: 1, External: 1.

Realisation

Finding an exploitable cycle is the same as finding a negative edge weight cycle in the graph of possible inventory states, where edge values represent how much money you lose in each trade (and negative for gains)

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Finding an exploitable cycle is the same as finding a negative edge weight cycle in the graph of possible inventory states, where edge values represent how much money you lose in each trade (and negative for gains)

We can run a sort of binary search / Bellman Ford mix to find the shortest negative cycle in this graph.

Make sure to use fast matrix exponentiation to fit within time bounds!

Binary Runs - 30 Points

Author: Jackson, Monash: 0, External: 0.

Main things to compute:

$$\Pr(b, a) := \text{Probability of jumping from binary string } b \text{ to } ba.$$

$$L(b) := \text{Length of last run on } b.$$

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Realisation

$$f\left(\frac{p}{q}\right) = pq^{-1} \text{ satisfies}$$

$$f(a+b) = f(a) + f(b)$$

$$f(ab) = f(a)f(b)$$

$$f\left(\frac{ap}{aq}\right) = f\left(\frac{p}{q}\right)$$

Binary Runs - 30 Points

Author: Jackson, Monash: 0, External: 0.

Include the probability of game ending in FailAdd, SuccessFlip etc.

Example Calculation

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Example Calculation

$$x := \text{FailAdd}(L(b0))\text{SuccessFlip}(L(b0))$$

$$y := \Pr(b0, 0) + \Pr(b0, 1)\text{SuccessFlip}(L(b0))$$

$$\begin{aligned}\Pr(b00, 0) &= y \times \left(\frac{\text{SuccessAdd}(L(b0))}{2}\right) \times \left(1 + x + x^2 + \cdots\right) \\ &= y \times \left(\frac{\text{SuccessAdd}(L(b0))}{2}\right) \times \left(\frac{1}{1 - x}\right)\end{aligned}$$

Similar idea for ending with 01, 10, 11.

Safe Exploring - 30 Points

Author: Jackson, Monash: 3, External: 6.

Realisation

We don't need to solve the queries in order!

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Realisation

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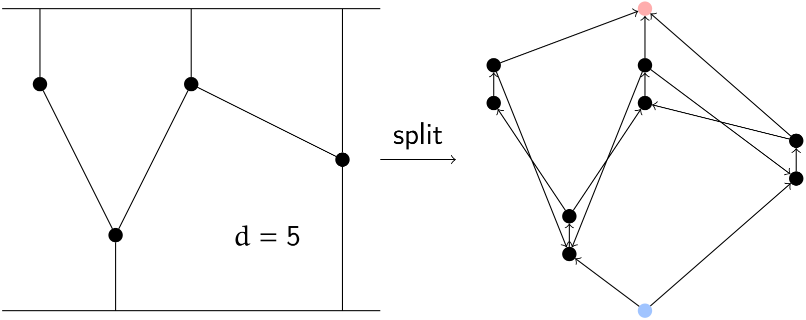
So order the queries based on increasing s, and use a Union Find to compute component sizes.

Scuttlebug Jamboree - 30 Points

Author: Jackson, Monash: 0, External: 3.



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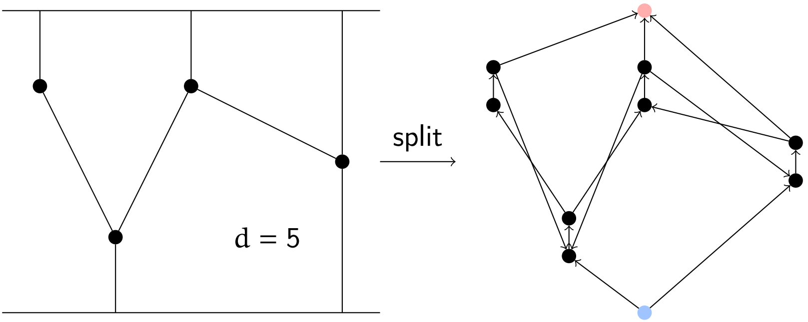

Realisation

We need the max-flow through the auxiliary graph to be $2b$ or more. So binary search on d to find the smallest value satisfying.

Government Divided - Part 2 - 100 Points

Author: Jackson, Monash: 1, External: 3.

`DP[x][k]` := How many distinct trees divide x work over k steps?

`DP[x][k]` with `k=0` is always 1, since we don't do any work division.

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Realisation

The number of divisions of x work over k steps, where the first step is breaking work up into y work (a divisor of x) sections is equal to

$$\binom{\text{DP[y][k-1]} + \text{x/y} - 1}{\text{DP[y][k-1]} - 1}$$

Stars and Bars - decide how many of each of the DP[y][k-1] options for subtrees we see. With DP[y][k-1] = 4, and x/y = 6:

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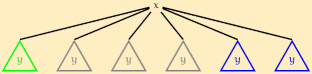

Hipster Train - 100 Points

Author: Jackson, Monash: 0, External: 1.

Idea

A lower capacity is always better.
Let `DP[i][h]` := After reaching station `i` and loading `h` hipsters over the course of the trip, what is the minimum number of passengers currently boarding the train?
Then answer is largest `h` for which `DP[n][h]` exists.

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Each `DP[i][h]` can then be calculated by looking at `DP[i-1][h-j]`. This is equivalent to picking up `j` hipsters at station `i`. But this is $10^3 \times 10^4 \times 10^4$. Way too long!

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Rather than picking up 1 hipster, then 2, then 3 and so on, pick up 1 hipster, then 2, then 4, then 8, ... Since every number in-between has a binary representation, we are catching these cases (Make sure to update the DPs in order of decreasing h however!)

Marble Swapping - 100 Points

Author: Jackson, Monash: 0, External: 0.

Realisation

The state space is smaller than it seems!

- Remove Wild Marble
- Ignore marble ordering

$$7^{14} \text{ vs } \binom{11}{5}^2 \approx 2 \times 10^5$$

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Each vertex has max-degree 36 (Often smaller). So start at winning/losing states, and search backwards in time. Marking states with a winning move winning, and marking states with all losing moves losing. Then answer all queries on constant time.

Connected Fungus - 100 Points

Author: Jackson, Monash: 0, External: 1.

We have a rooted tree. Use an LCA for quick ancestor calls.

Realisation

$$d(i,j)^2 = (\mathrm{dep}[i] + \mathrm{dep}[j] - 2 \times \mathrm{dep}[\mathrm{lca}(i,j)])^2$$

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Main idea: Calculate all interesting days, where something spawns or dies. Keep a track of the current work per day, and change this as things live or die.

- How does a spawned vertex affect the sum of $d(i,j)^2$?
- How does a removed vertex affect the sum of $d(i,j)^2$ (It also disconnects all sub-trees)

Connected Fungus - 100 Points

Author: Jackson, Monash: 0, External: 1.

Store:

- $n_l :=$ Number of alive nodes below sub-tree l
- $d_l :=$ Sum of dep[i] below sub-tree l
- $s_l :=$ Sum of dep[i]^2 below sub-tree l

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- s_l := Sum of dep[i]^2 below sub-tree l

Let l be the highest alive ancestor of i:

$$\begin{aligned}\sum_j d(i,j)^2 =& n_l \mathrm{dep}[i]^2 + 2\mathrm{dep}[i] d_l + s_l + 4\left(\sum_j \mathrm{lca}(i,j)^2\right) \\ & - 4\mathrm{dep}[i]\left(\sum_j \mathrm{lca}(i,j)\right) - 4\left(\sum_j \mathrm{dep}[j]\mathrm{lca}(i,j)\right)\end{aligned}$$

This if for spawning a new vertex i. How can we compute those sums quickly...

Results!